

KANT ON MATHEMATICAL AXIOMS¹

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INTRODUCTION

In 1762, Immanuel Kant wrote a text for the competition of the Berlin Academy of Sciences. In order to participate in this competition, philosophers had to pronounce on the possibility of achieving in metaphysical research the kind of certainty that mathematical knowledge had already conquered. Our philosopher approached the subject from a methodological perspective and developed some aspects of a thesis that he would revisit in the *Critique of Pure Reason*, almost twenty years later³. According to this thesis, the procedures that are successfully employed in mathematical research are inconducive in the domain of metaphysics. This thesis is developed in detail in the section entitled “The Discipline of Pure Reason in its Dogmatic Use” of the 1781 text. For Kant, the success achieved by mathematics in its investigations is based on the procedures that lead it. These procedures include the use of definitions, axioms and demonstrations. The philosopher argues that in metaphysics none of these three elements is feasible *as mathematics conceives them*⁴.

The starting point of his explanation is an insight into the difference between mathematical and metaphysical practice. The mathematician obtains knowledge through the construction of concepts in intuition. The philosopher, on the other hand, proceeds discursively and knows through mere concepts. This way of approaching the difference between the two disciplines, which does not reduce it to a difference between knowing quantities and knowing qualities⁵, is the ground on which the thesis of the methodological difference between the two is based. As we have pointed out, this is supported by the fact that mathematical knowledge proceeds by definitions, axioms and demonstrations, while none of these is feasible in the case of rational discursive knowledge, in the sense in which mathematics embraces them.

With regard to definitions, it is noticeable that Kant develops a strict sense of this term, which is the one he finds, in particular, at the beginning of mathematical research. A definition

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is a concept that has certain specific perfections. In particular, it is a concept that is distinct, detailed, original and set within precise limits. The clarification of concepts that takes place in metaphysical research, or in propaedeutic research for metaphysics such as we find in the critical text, constitutes a good and fruitful explanation of such concepts. However, this philosophical approach deals with already given concepts, so that it is neither exhaustive nor original. Kant denominates the way of approaching concepts that can take place in metaphysical research as “exposition”⁶.

Secondly, mathematical research includes axioms. In the sense in which we understand this term in the field of mathematics, axioms, according to Kant’s *Critique*, are synthetic a priori judgements characterised by immediate certainty. In connection with this definition of the term “axiom”, we have to explain why it is inconceivable for Kant that cognition by mere concepts is not capable of attaining immediate certainty, unlike mathematical cognition which proceeds by construction, and why philosophy does not contain axioms.

Finally, Kant refers to the notion of demonstration. For Kant, a demonstration is an exhibition in intuition. Our philosopher recovers the etymological sense of the term. On the one hand, in Kant’s texts, one notices the difference between the mathematical, technical sense of this term, which is explained in the First Critique, and the empirical sense, associated with the practices of the natural sciences, which is outlined in the *Critique of Judgement*. In relation to the procedure proper to mathematics, finally, it is possible to identify two problems. On the one hand, it can be discussed whether there are demonstrations in all disciplines of mathematics or whether this procedure is exclusive to geometry. On the other hand, the relationship and the differences between a demonstration and the construction of a concept must be explained⁷.

In this article we examine the notion of axiom, in a mathematical sense. To do so, firstly, we review the definition of the term that can be found in the mathematics texts that Kant used in his courses. Secondly, we will dwell on some brief and marginal appreciations that we find in the pre-critical sources, and which are considered to be a sort of work in progress by our philosopher. To examine Kantian views on criticism, we first look at some notes from his logic lectures, and then at the relevant passages in the *Critique of Pure Reason*. Finally, we discuss the notion of the “axioms of intuition”.

The present study stops at that point. The evolution of Kantian thought on axioms does not. In a future paper, we hope to consider the debate with Eberhard, the epistolary with Schultz, and Kant’s use of axioms in the construction of his practical philosophy. In this article we will only explain the peculiarity of the critical view of mathematical axioms, by means of a comparative study of this view, the pre-critical development and the Wolffian insight into these propositions.

DEFINITIONS OF “AXIOMS” PROVIDED BY MATHEMATICIANS

One of the philosophers who must be considered in the first place, if we want to examine the peculiarity of Kant's positions on the mathematical method, is Christian Wolff. There are several reasons for this. On the one hand, Wolff was considered the main supporter of the thesis of the methodological monism of mathematics and philosophy, the discussion of which is the axis around which Kant developed his remarks on axiomatics⁸. Secondly, from Kant's earliest writings on the subject Wolff is presented as the explicit adversary of the philosopher from Königsberg⁹. On the other hand, his texts were the source for the mathematics courses that Kant taught in his youth¹⁰. We will rely especially on the latter reason for considering Wolff's texts as a source for understanding the mathematics of the time¹¹.

The books that Kant used in his lessons are entitled *Der Anfangs-Gründe aller mathematischen Wissenschaften*, published in Frankfurt/Leipzig in 1710, and *Auszug aus den Anfangsgründe aller matematischen Wissenschaften*, published in Halle in 1713. The former has four volumes, is longer and, perhaps for that reason, unsuitable for use in courses¹². Kant would have chosen to use the *Auszug* for his lectures and this text would be the reference for Herder's notes, which are the transcriptions of the lectures that are currently available and with which we will work here.

In relation to the issue we are interested in, there are no obvious differences between the two texts. We are concerned here with the opening paragraphs, which are found after the prologue, in a section entitled: “Kurtzer Unterricht/ Von der Mathematischen Methode” (1710) and “Kurtzer Unterricht, von der Mathematischen Lehr-Art” (1713). In §1, Wolff states the following: “The doctrine (*Lehrart*) of the mathematician begins with definitions (*Erklärungen*) and proceeds to axioms (*Grundsätzen*), and from them to theorems (*Lehrsätzen*) and problems (*Aufgaben*)”¹³. In the *Auszug*, Wolff makes it clear that doctrine is the order he uses in his contributions¹⁴. Immediately after this statement, the philosopher explains each of the elements contained in it.

In *Der Anfangs-Gründe Aller Mathematischen Wissenschaften*, Wolff explains that principles are obtained directly from definitions, whether they are definitions of words or of things. Thus, for example, from the definition of the circle I can know that all lines from the centre to the circumference, i.e. all *radii*, have the same length. This is a necessary truth and is known from the mere definition of the circle. For Wolff, it constitutes a principle (§ 27). A principle teaches how something is or that something can be done. Principles that refer to the characteristics of things are called *axiomata*, while principles that refer to a possible act are called *postulata* (§28).

Now, Wolff adds a further feature of these principles. Since they are immediately derived from definitions, they do not require any proof. The truth of them becomes clear as soon as we attend to the definitions from which we have derived them. Once we find such definitions, and only in this way, can we know whether the principles we derive from them are true. In other words, the truth value of these propositions is guaranteed by our definitions. But it is necessary to have them in order to be able to determine it (§29)¹⁵.

This way of understanding the axioms is repeated in other texts by the professor from Halle. Thus, for example, we can see it in the *Mathematisches Lexicon*, written and published a few years later. According to this text, an axiom or principle (*Grundsatz*)¹⁶ is a proposition so clear that one admits it without proof. As in the 1710 text, Wolff quotes here Professor Tschirnhaus, who in his treatise *Medicina Mentis* (1687) calls propositions whose truth (*Richtigkeit*) is made clear from some definition “axioms”. For Wolff, however, it is necessary to specify the peculiar use that, in particular, the term has had in mathematics since Euclid¹⁷.

In his *Auszug* on Mathematics, which is a sort of summary of the 1710 book, Wolff develops this question in greater detail. According to Tschirnhaus’ definition, a principle (*Grundsatz*) is derived from a definition, whether of a word or of a thing, such that by examining its content something else immediately follows from it. In mathematics, however, we understand by “principle” a general proposition which is admitted without proof. Among such principles we may distinguish those which show what something is from those which show that something can be done¹⁸. The former are called *axioms*; the latter, *postulates*¹⁹.

According to the texts we have taken into account, for Wolff the axioms of mathematics are principles that state that something is a certain way, are obtained immediately from definitions and do not need proof. They are regarded as undisputed truths and constitute fundamental propositions for knowledge. It is worth noting the feature of axioms that means that they do not have to be proved. In the passages under consideration, it is pointed out that this is because they follow from definitions. In the *Lexicon*, a complementary interpretation is presented, which suggests that axioms need not be proved because of self-evidence²⁰. In other passages, especially in the logic texts, the axioms are presented as “*propositiones identicae*”²¹. It has been explained in other research that this description is complementary to that found in writings on mathematics. In these writings, indeed, the evidence of axioms is supported by the fact that they follow from certain definitions. In this sense, they constitute propositions that become clear as soon as we know the concepts clearly and distinctly²².

KANT’S PRE-CRITICAL REMARKS ON AXIOMS

In the currently available pre-critical documents, we find few, albeit significant, references to the notion of “axiom”. First of all, we find a reference in a note from the 1960s based on mathematics lectures given by Kant. We have two sets of notes, NL-Herder, Ms XXV, 45 and 46. In both cases, the annotations, which would have been taken between 1762 and 1764, are brief and not very articulated. There are, however, some interesting indications about the nature of the axioms. These are presented in the context of an examination of mathematics, which reviews the nature of its object and its manner of arguing and exhibiting arguments. According to Herder’s notes, Kant commented on the notion of axioms in his lessons. According to this testimony, axioms were described in these lectures as theoretical propositions (*Sätze*) which are not proved (*unerweislich*) and, in this sense, as principles (*Grundsätze*) or common notions²³.

In the text *Untersuchung über die Deutlichkeit der Grundsätze der natürlichen Theologie und der Moral*, written in 1762 and published in 1764, we can find some references to the procedures of mathematics. In relation to the topic at hand, in this text Kant points out that the mathematical sciences are based on a few unproven propositions which are considered to be immediately true. The philosopher mentions in this text two examples, which say (i) that the whole is equal to the total of its parts and (ii) that between two points we can only draw a single straight line. As we shall see, this second example will be a favourite of Kant in the *Critique*. Moreover, he adds that mathematicians declare these principles undemonstrated at the beginning of their investigations and thereby establish that only self-evident propositions are assumed to be true and that everything else is rigorously proved²⁴.

In texts of the following decade, the explanation of the concept in question is more detailed and better linked to its critical version. Moreover, there are some novelties with respect to the Wolffian view. In the *dissertation* of 1770, Kant does not explain the concept but uses it in a peculiar way. He says that in natural philosophy and mathematics the axioms are given by intuition, whereas the axioms of metaphysics are given by pure understanding and are, for this reason, liable to error²⁵. As we shall see, in the remaining testimonies of this decade and until the publication of the *Critique of Pure Reason*, he will reject the idea that there are axioms in metaphysics, or in discursive knowledge in general.

In a reflection from the beginning of the silent decade, to begin with, we read that axioms possess a primitive certainty²⁶. In another annotation by Kant, written sometime in the same decade, axioms are defined as “*iudica intuitiva a priori*”. In the same note, it is pointed out that analytic discursive judgements (such as the principle of contradiction) do not qualify as axioms. And among the synthetic principles, only the intuitive ones constitute axioms. For this reason, philosophy, whose principles are acroamatic, has no axioms²⁷. In the same way, in a reflection on metaphysics of the same decade, axioms are also presented as a class of synthetic propositions that are not proved²⁸. Likewise, in a reflection of the second half of that decade, Kant defines an axiom as an “immediately true intuitive a priori judgement”²⁹. These annotations are found in the margins next to §315 of the *Auszug aus der Vernunftlehre*, with which our philosopher taught Logic. In Meier’s text one reads, along the same lines as in Wolff’s mathematical texts, that axioms or fundamental judgements (*Grundurteile*) are not proved.

If we look at the annotations of Kant’s students, in the currently available annotations on the *Encyclopaedia philosophica*, dating from the second half of the silent decade, it is pointed out that metaphysics will not provide dogmatic propositions or a priori axioms³⁰. Then, it is added that mathematics has axioms, i.e. “*a priori principles of intuition*”, whereas philosophy has only principles for discursive a priori knowledge³¹.

Thus, the study of the pre-critical sources allows us to notice an evolution in the approach to the axioms. Before the *Disertatio*, Kant broadly repeats the standard Wolffian view. In the silent decade he seems to be discovering the properties that he will attribute to the axioms in the *Critique*. They are presented, in the sources of the “silent decade”, as non-proved, self-evident propositions, constituting synthetic a priori judgements and containing self-evident truths.

Moreover, if in the dissertation the possibility of certain philosophical axioms is suggested, later annotations will restrict the sphere of axioms to the realm of mathematics.

THE NOTION OF “AXIOMS” IN THE EARLY 1780s

In the lectures on logic of the early 1780s, axioms are described as immediately certain knowledge that is assumed to be so because it can be known a priori from the nature of the thing³². In Hechsel's Lecture-Notes on Logic, dated 1782, the notion of judgements that we cannot prove is explained. In particular, we read that there must be some unprovable judgements that are immediately true. The example is the one that says that between two points we can only draw a straight line.

Among the judgements that are not proved, we find some judgements based on identity. Those that assert an identity explicitly, such as the judgement that says: “Human beings are human beings”, are empty analytical judgements, self-evident and do not require proof. Other judgements are based on the principle of identity in an implicit way, as for example the judgement that says: “Human beings are rational animals”. This judgement is analytical, but explanatory. The analysis of the concept of the subject allows us to find in it the concept of the predicate. This analysis constitutes a proof of the judgement, which thus does not constitute a mere empty and self-evident proposition.

Among the judgements that are not proved, in addition to the empty analytical judgements, certain synthetic judgements are mentioned, such as those of geometry that we have previously dealt with. Judgements that are not proved, whether analytical or synthetic, are called “principles”, can be known a priori and are not based on other judgements, but are themselves the ground for other judgements³³. Among the principles, these Lecture-Notes differentiate between those which are intuitive and are called axioms, and the discursive principles of philosophy, for which there is yet no name. In these notes, the name “acroama” is proposed for the latter. Finally, these principles are distinguished from postulates, which teach what is to be done³⁴.

THE EXPLANATION OF AXIOMS IN THE *CRITIQUE OF PURE REASON*

We can find numerous references to the notion of axioms in the text of the first Critique (=KrV). First of all, in the “Transcendental Aesthetics” Kant refers to the axioms of time, such one that states that time has only one dimension. The philosopher points out that these are apodictic principles. They cannot be grounded in experience, since experience is neither a source of apodictic certainty, nor of strict universality. Now, as Kant teaches in the Introduction to the KrV, apodictic certainty and strict universality exhibit that a cognition has a priori principles. Thus, the axioms of time have an a priori origin and are a foundation of all possible experience for us³⁵.

In the *Methodenlehre*, particularly in the section entitled “Discipline of Pure Reason in its Dogmatic Use”, the explanation of the concept includes some additional determinations. Axioms, Kant states, are synthetic a priori judgements that are self-evident, i.e., that involve immediate certainty. This latter feature excludes the principles of philosophy from the set of axioms and implies that such philosophical principles, unlike mathematical ones, require deduction³⁶. In addition to immediate certainty, this explanation of axioms includes a relevant property: they are synthetic judgements.

Thus, we see that the axioms, as presented in the KrV, are a priori, synthetic and self-evident judgements. Excluded from this set are thus empirical judgements, analytic judgements and judgements whose certainty is mediate. This last feature is central to the argument of the *Methodenlehre*. The set of axioms excludes those judgements that link a predicate already contained in the concept of the subject, i.e., analytic or explanatory judgements. The synthetic a priori judgements are those of mathematics, such as the one that says that three points share a plane, and those of philosophy, such as the one that says that whatever happens has its cause. For Kant only the former are axioms by virtue of the fact that, as we have pointed out, the connection that such judgements express is immediately evident. The connection arises in intuition, when we construct the concepts of mathematics.

These mathematical judgements constitute principles for mathematical knowledge. This is because they do not require proof, since they are supported by evidence, and are the starting point for the construction and demonstration of other judgements. In the introduction to the “Transcendental Dialectic”, Kant again refers to the axioms of mathematics in order to specify the sense in which they constitute principles. He mentions in this case his most usual example, according to which we can only draw a straight line between two points. He further characterises them as “a priori universal knowledge”³⁷ and examines the sense in which they can be regarded as principles (*Principia, Principien*)³⁸. Kant is explaining what it means that reason is the faculty of principles. In this context, he points out that in mathematics we do not cognise from principles. To know from principles is to know the particular in the universal through concepts. In mathematics we do not know through concepts, but by construction in pure intuition. The mathematician knows by constructing in intuition and proves from judgements which are self-evident by virtue of the nature of the pure form of our intuition.

Now, in the “Analytic of Principles”, Kant presents an additional determination of axioms, which allows us to differentiate between two concepts: axioms in the broad sense and axioms in the strict sense. In the broad sense, axioms are synthetic a priori self-evident judgements. Evidence implies that these judgements must be intuitive, because the certainty about them is immediate. Discursive principles are not self-evident, they require deductions. As we have already noted, only in mathematics do we find such principles. Kant even claims that the knowledge provided by philosophy could not be as self-evident as the proposition that says: “ $2+2=4$ ”³⁹.

Now, as we have already seen, in the “Analytic of Principles” Kant also develops a way of defining axioms according to which only geometry includes such principles. Kant mentions two examples, again. These are the judgement that we can only draw a straight line between

two points and the judgement that we cannot enclose a space with two straight lines⁴⁰. These judgements, according to the philosopher, express conditions of a priori sensibility that make the construction of figures in space possible.

The additional determination that leaves the judgements of arithmetic outside the set of axioms is that of their universality. Kant notes that arithmetic has numerous synthetic, immediately true propositions. However, these propositions do not have universal scope, but are singular judgements. Their content is a unique way of synthesising the homogeneous. Kant proposes to call them “*formulae*”. Arithmetic contains analytic judgements, such as the one that an equality added to another equality results in a (third) equality. Formulas are not such analytical judgements, but, like axioms, contain a synthesis. The example the philosopher mentions is that of any sum, such as “ $7+5=12$ ”. Kant explains that the synthesis contained in this judgement can be realised in only one way and that if a formula were an axiom, then we would have infinitely many axioms⁴¹. Briefly, arithmetical formulae are excluded from the strict sense of axiom, which is no longer mentioned after the distinction made in this passage.

THE “AXIOMS” OF INTUITION

The pure principles of understanding are presented as synthetic a priori judgements that constitute a foundation for other judgements and are not themselves founded on others. Up to this point, their characteristics coincide with the synthetic a priori judgements that are the axioms of mathematics. However, as we have remarked, mathematical axioms are presented as immediately certain. The principles contained in critical enquiry, on the other hand, are characterised by the fact that they are not exempt from proof. The kind of proof they require, in particular, is based on the examination of the subjective sources that make knowledge in general possible⁴².

For Kant, the investigation of the principles of understanding is crucial in virtue of the fact that they refer to the conditions of possibility of all possible experience for us. In this sense, also the possibility of mathematics being referred to experience and having objective validity rests on these principles. The principles of understanding function as principles for mathematics. In the system of pure principles of the intellect, Kant calls “axioms of intuition” those that refer to the categories of quantity⁴³. This is the reason why we deal with some aspect of them in this work.

Kant notices in the *Critique* that these “axioms of intuition” do not constitute principles of mathematics. The principles governing mathematics are determined by the pure forms of our sensibility. It is by virtue of the nature of spatial form in general that two straight lines cannot enclose a space, or that we can only draw a straight line passing through two points. It is because they are based on our way of intuiting that such principles are self-evident. The pure principles of the understanding, on the other hand, refer to the application of the pure concepts of the understanding to a possible experience and, in this sense, are based on the nature of our understanding. The synthesis referred to in the axioms of intuition applies to

the form of our intuition. Since this form necessarily extends to all phenomena that are to constitute our experience, the corresponding principle is stated unconditionally. The certainty attained in its application is intuitive. It is by virtue of their application, and not by virtue of their origin⁴⁴, that Kant regards these principles as mathematical and calls them “axioms”.⁴⁵

Kant emphasised that the principles of quantity studied in the “Analytic of Principles” are not axioms. In the “Discipline” he expressly points out that the principle is not itself an axiom, that it is a principle which makes it possible to understand the possibility of axioms and that it itself is derived from concepts. The principles of philosophy, based on the concepts of our intellect, must be deduced⁴⁶. Such deduction shows how mathematics is possible, but it does not belong to the proper domain of this science. In his study devoted to this section of the *Critique*, Oliver Schielmann points out an additional reason for not identifying the “axioms of intuition” with those of mathematics. The author notes that the principle of the “axioms of intuition” is not to be found in any of the known books on mathematics and adds that the scope of the principle of Kantian analytics is greater because it not only assures the objective validity of mathematics but also refers to a determination of all possible intuitions for us. In this sense, the principle is a judgement of philosophy and not of mathematics⁴⁷.

In short, Kant does not identify the principle presented in the “Axioms of Intuition” with the geometrical axioms. It seems that the axioms of intuition provide some kind of ground for geometrical knowledge. The main reasons why several Kantian scholars have endorsed this interpretation are the following⁴⁸. On the one hand Kant states in the “Discipline of pure reason” that the *Grundsatz* only serves to supply the principle for the possibility of axioms⁴⁹. On the other hand, from some available Kantian it follows that the *Grundsatz* is a condition of the possibility of applied mathematics⁵⁰. In addition, in the last paragraph of the chapter, Kant states that “this transcendental principle of the mathematics of appearances... alone makes pure mathematics applicable in all its precision to objects of experience”⁵¹. In the same vein as in the present article, Daniel Sutherland points out that “these principles are not themselves mathematical but are called mathematical because they explain the possibility of mathematical principles (A162/ B201–2). The mathematical principles whose possibility is explained include geometrical axioms, such as Euclid’s postulates. Nevertheless, the Axioms of Intuition argue for just one principle that is not itself an axiom”⁵².

CONCLUSION

In this article we have tried to elucidate which is the mathematician’s notion of axiom that is impossible for Kant to have in the realm of philosophy. To do so, we began by examining the definitions of one of the mathematicians Kant consulted, namely Wolff. Wolff defines axioms as principles of knowledge, which are obtained from definitions, are self-evident and need no proof. We have seen that Kant maintains this definition until his dissertation. But from then on, he modifies it significantly. For our philosopher, in his critical period, axioms are defined as synthetic a priori self-evident judgements. Evidence is the feature that makes

them possible only in mathematical research, since the judgements of this science are based on the pure form of our intuition. We have further reviewed two issues that are often discussed in connection with this critical view of axioms. On the one hand, we examined the narrow meaning of “axiom”, which excludes the of arithmetic. On *formulae* the other hand, we have lingered on the choice of this term to name the principle corresponding to the category of quantity. In connection with this point, we have argued that this principle is not an axiom, but it does serve as a basis for justifying the objective validity of the science of axioms, which is mathematics.

Abstract: This article is intended to explain the notion of “mathematical axioms” presented in Kant’s *Critique of Pure Reason*. This notion is developed mainly within the framework of a justification of the thesis of the methodological dualism of the rational sciences (mathematics and metaphysics). We argue that there are significant differences between the critical notion of mathematical axioms, the pre-critical developments and the Wolffian definitions. The notion of “axiom” that Kant intends to take from mathematical procedures is inscribed in his peculiar way of thinking this science. This paper studies the considerations of (i) Wolff’s mathematical texts, (ii) the pre-critical texts and (iii) the *Critique of Pure Reason*, and mentions the differences between them in the conclusion.

Keywords: Kant; Wolff; Axioms; Mathematics; Criticism.

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NOTES/NOTAS

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³Cf. Martínez 2018.

⁴KrV A726, B754.

⁵A more detailed presentation of this thesis can be found in Sgarbi 2010, 239f.

⁶Cf. Martínez forthcoming.

⁷Cf. Martínez 2022.

⁸Cf. Pelletier 2005.

⁹Cf. *Deut*, AA 02: 277.

¹⁰Cf <https://users.manchester.edu/facstaff/ssnaragon/kant/Lectures/lecturesTextbooks.htm>

¹¹Along the earlier suggestions, moreover, by Engfer (1982, 220f.).

¹²Prolific, Wolff published several texts in which he explained the fundamentals of mathematics as this science was understood in his time. In addition to the texts used by Kant and mentioned here, there is a "lexicon" of mathematical terms which we will also take into account in this work.

¹³Wolff, 1710: 5.

¹⁴Wolff, 1717: 1.

¹⁵Wolff, 1710: 14s.

¹⁶According to the Brothers Grimm dictionary, "Grundsatz" is Wolff's translation of "axiom". Cf. "GRUNDSATZ, m.", *Deutsches Wörterbuch von Jacob Grimm und Wilhelm Grimm*, Bd. 9, p. 890, l. 35.

¹⁷Wolff, 1716:223s

¹⁸Shabel 2003: 50.

¹⁹ Wolff 1755, 5.

²⁰ I stress this point because Engfer, in a more exhaustive and detailed work of research than the one presented here, concludes that Wolff follows the Leibnizian tradition which grounds the fact that axioms do not have to be proved on the fact that they are immediately inferred from definitions, contrary to the interpretation, which, in the line of Proclus, supports this feature in the self-evidence of axioms and the fact that they do not derive from anything else. Cf. Engfer 1982, 233.

²¹ *Lógica latina*, §214. Ctd. in Schenk, G., Meyer, R 2009: 59.

²² Engfer 1982, 234.

²³ V-Math/Herd, AA 29: 51.

²⁴ *Untersuchung*, AA 02:281.

²⁵ *Diss*, AA 02:410f.

²⁶ R 4675, AA 18: 649.

²⁷ R 3132, AA 16: 672f.

²⁸ R. 4928, AA 18:31.

²⁹ R. 3135, AA 16:673.

³⁰ PhilEnz, AA 29:36.

³¹ PhilEnz, AA 29:38.

³² V-Log/Wien, AA 24:886.

³³ In Warschauer's notes on logic from the same period, it is added that judgements that cannot be proved are elementary propositions. Kant 1998, 630.

³⁴ Kant 1998, 436ff.

³⁵ KrV A31, B47.

³⁶ KrV, A732, B 760.

³⁷ KrV A300, B356.

³⁸ In all the other passages, when we use the word "principle", in German it reads "*Grundsatz*".

³⁹ KrV A733, B761.

⁴⁰ KrV A 163, B 204.

⁴¹ KrV A165, B 205.

⁴² KrV A148s, B188.

⁴³ H. Heimsoeth has indicated early on that strictly speaking the "axioms of intuition" include only one principle. Cf. Heimsoeth 1966, 682.

⁴⁴ Schliemann also notices this when he states that the axioms of intuition refer to intuition but are not inferred (*abgeleitet*) from it. Cf. Schliemann 2010: 202.

⁴⁵ KrV A162, B201.

⁴⁶ KrV A733, B761.

⁴⁷ Schliemann 2010: 99.

⁴⁸ Firstly, Mellin: „Sie [the axioms] sind wahre Axiomen [...], nemlich die Axiomen der Mathematik", p. 452.

⁴⁹ KrV, B761.

⁵⁰ Cfr. R. 5585, 5589.

⁵¹ KrV, B206.

⁵² Sutherland, 2005: 136.

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